An improved algorithm for computing the median of the Erlang distribution

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Resumen

Let $X_n$ be a random variable having the Erlang distribution with shape parameter $n + 1$ and scale parameter 1, that is, its cumulative distribution function is

$$F_n(x) := P(X_n \leq x) = \frac{1}{n!} \int_0^x t^n e^{-t} dt, \quad x \geq 0 \quad (n = 0, 1, 2, \ldots).$$

The median $\lambda_n$ of $X_n$ is defined as the solution of the equation $F_n(\lambda_n) = 1/2$.

Choi [2] gave a procedure for computing the following asymptotic expansion of $\lambda_n$

$$\lambda_n = n + \frac{2}{3} + \sum_{i=1}^r \frac{q_i}{n^i} + O\left(\frac{1}{n^{r+1}}\right), \quad (1)$$

where $q_i$ are rational coefficients, and he gave the first four terms. However, from a computational point of view, Choi’s procedure is not efficient and only a few terms can be obtained. Adell and Jodrá [1] gave the first seven coefficients and proposed a method to obtain rational bounds for $\lambda_n$ from expansion (1).

In this work, we develop an improved algorithm for computing a large number of terms in the asymptotic expansion of $\lambda_n$. In particular, we provide the exact values of the first sixty coefficients in expansion (1) which allow us to answer the following questions: (i) Does the series $\{|q_i|\}_{i \geq 1}$ converge to 0 such as numerical results in [1] may be suggesting? (ii) Which is the partial sum in (1) closest to the value of the median of $X_n$ as well as the number of correct significant digits?

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Bibliography
