Techniques for temporal dynamics of neuronal systems: the Hindmarsh-Rose model

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Abstract

A phenomenological system of ODEs proposed by Hindmarsh and Rose [2] for modeling bursting and spiking oscillatory activities in isolated neurons is given by:

\[ \begin{align*}
\dot{x} &= y - ax^3 + bx^2 - z + I, \\
\dot{y} &= c - dx^2 - y, \\
\dot{z} &= \varepsilon(s(x - x_0) - z);
\end{align*} \tag{1} \]

here, \( x \) is treated as the membrane potential, while \( y \) and \( z \) describe some fast and slow gating variables for ionic currents, respectively. Slow “activation” of \( z \) is due to the small parameter \( 0 < \varepsilon \ll 1 \). The parameters in (1) are typically set as follows: \( a = 1 \), \( c = 1 \), \( d = 5 \), \( s = 4 \), \( x_0 = -1.6 \) and \( \varepsilon = 0.01 \), so that regular bursting oscillations in the model at an “applied current” \( I = 4 \), which belongs to the square-wave type at \( b = 2.7 \), and transforms to a plateau-like bursting at \( b = 2.52 \). Along with “intrinsic,” \( b \), and “external,” \( I \), bifurcation parameters the dynamics of the model are sensitive to variations to other parameters: \( \varepsilon \) being treated as a rate of activation for some current, and \( x_0 \) being viewed as a control parameter delaying and advancing the activation of the slow current in the modeled neuron.

In this talk we analyze metamorphoses of oscillatory activities, such as plateau-like and square-wave bursting in the Hindmarsh-Rose model by using a complementary suite of computational tools developed for two-parameter screening of dynamics in neuronal models [1]. The comparison involves “calculus” based tools for the evaluation of an entire spectrum of Lyapunov exponents commonly employed in bifurcation studies, with neuroscience plausible techniques specifically tailored for the fast examination of temporal characteristics of neural activities including spike counting, duty cycle of bursting, interspike interval, etc.

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Bibliography
