Global study of the Bogdanov-Takens bifurcation curve

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Resumen

The bifurcation curve of the homoclinic connection that appears in the Bogdanov-Takens normal form system

\[
\begin{align*}
x' &= y, \\
y' &= -n + by + x^2 + xy,
\end{align*}
\]

is usually restricted to a local study near the critical point. The aim of this work is to obtain a global knowledge of this curve in the parameter space.

It is well known that for small \( n > 0 \) there exists a value \( b^*(n) \) such that the system has an unique limit cycle if and only if \( b^*(n) < b < \sqrt{n} \). Moreover \( b^*(n) = \frac{5}{4} \sqrt{n} + \ldots \). In [3] the quadratic differential equation is considered in the whole space and it is proved that \( b^*(n) \) is analytic and that the previous inequalities are satisfied for all \( n > 0 \). A detailed study of the curve \( b^*(n) \) for\( n \) small enough is presented in [2]. This previous work is based on an algebraic method for the location of bifurcation curves.

In our work, see [1], we adapt this procedure to the global study of the Bogdanov-Takens bifurcation. That is, we obtain explicit curves such that \( b_l(n) < b^*(n) < b_u(n) \) for all \( n > 0 \). With this result we proved a Perko’s conjecture, see [3], that predicts the behaviour of this curve when the parameters goes to infinity. In fact, we proved that \( b^*(n) \) goes to infinity as \( \sqrt{n} - 1 \).

Sección en el CEDYA 2011: EDO

Bibliography

