Monetary policy regimes
and the forward premium anomaly: A note (*)

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Abstract: This paper provides a theoretical discussion of the forward premium anomaly. We reformulate the Dutton model (Dutton, 1993) by allowing the existence of monetary policy regimes. The monetary supply is viewed as having two stochastic components: a) a persistent component that reflects the preferences of the central bank regarding the long-run money supply or inflation target, and b) a transitory component that represents the short-lived interventions. To generate agents’ forecasts, we consider two scenarios: 1) consumers can distinguish the permanent and the transitory component and b) consumers can only observe historical series of the aggregate monetary supply and they face a signal-extraction problem. The model proposed can reproduce the forward premium anomaly. Our theoretical results show that “anomalous” or negative slopes tend to be higher under high and positive correlation between the transitory monetary shocks. Also, when monetary policy is implemented in two countries in a similar way, the number of negative slopes under rational learning clearly departs from the number that corresponds to complete information. Both scenarios are according to the countries analyzed in the literature.

Key words: forward premium anomaly, monetary policy, regime shifts, learning

JEL classification: C22;F31;F47.

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1. INTRODUCTION

According to the hypothesis of uncovered interest rate parity, expected changes in the spot exchange rates should be perfectly and positively correlated with the current forward premium. In that case, the domestic currency is expected to depreciate when domestic nominal interest rates exceed foreign interest rates. However, not only has this hypothesis been empirically rejected in a large number of studies (see, for example, Froot and Thaler, 1995, Lewis, 1995, Bansal and Dahlquist, 2000, Tauchen, 2001, Sercu and Vinaimont, 2006, among many others), but a negative correlation has also been detected. This empirical feature is known as the forward premium anomaly and has direct implications for expected returns from international currency deposits. For a given positive interest rate spread, higher negative correlations between exchange rate change and interest rate differentials imply a higher expected excess return.

This puzzle has been interpreted in several ways in the literature. From a theoretical point of view, although a substantial number of studies have addressed the ability of general equilibrium models related to the Lucas (1982) model to explain the forward premium puzzle (see, for example, Hodrick (1989), Macklem (1991), Canova and Marrinan (1993), Bekaert (1994), Engle (1996), Baillie and Ostenberg (2000)), they either require unreasonable risk aversion parameters or more volatile consumption processes than in reality. Relative to empirical work, recent studies focus on the non-stationarity and long-memory features of the exchanges rates and the forward premium (see, for example, Baillie and Bollerslev (2000), Tauchen, 2001, Maynard (2003, 2006), Maynard and Phillips (2001) and Maynard and Lu (2005), among many others). In sum, a good deal of empirical and theoretical work has been devoted to analyzing the forward premium anomaly, but a conclusive understanding of the issue continues to elude researchers. The anomaly is still regarded as one of the most important puzzles in international finance.

In this paper, we propose a theoretical general equilibrium model to explain the forward bias for foreign exchange. The model is based on the Lucas (1982) model and is
similar to that proposed in Dutton (1993). In a recent paper, Mark and Moh (2007) investigate the idea that the forward premium anomaly is caused by unanticipated central bank interventions in the foreign exchange market. These authors propose a theoretical model in which the violations to uncovered interest parity do not reflect unexploited profit opportunities or systematic risk. The numerical simulations of the model show that the forward premium anomaly intensifies during periods in which central banks are intervening. In accordance with Mark and Moh (2007), our model mainly departs from the Lucas model in that individuals can observe the historical sequence of money supply, but they cannot distinguish whether monetary shock implies a regime shift or a transitory intervention. Monetary policy is specified as having two components: one is determined at each time period by the prevailing monetary regime and reflects a particular target of the central bank regarding the money supply or inflation; the second represents short-lived interventions. Consequently, economic agents face a signal-extraction problem through a learning mechanism that allows breaking down monetary shocks into transitory and permanent components. Also, similarly to Lafuente and Ruiz (2006), the potential correlation between monetary shocks in the two countries is explicitly taken into account. Moreover, given that the model proposed allows the existence of real shocks, no restriction is imposed on the correlation between these shocks in the two countries. Consequently, a broad set of scenarios can be simulated in order to explore potential explanatory factors of the forward bias.

The remainder of the paper is structured as follows. Section 2 describes the theoretical model. Section 3 presents the analysis of the simulated data. Finally, Section 4 summarizes and provides concluding remarks.

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1 Based on the Lucas model, numerous studies in the literature attend either to the determination of exchange rates or to the forward puzzle (see, among many others, Campbell and Cochrane (1999) and More and Roche (2002)).
2 THE MODEL

The model we consider is basically the Lucas model, but we incorporate the existence of regime switches in the monetary policy. Two countries have their own currency and a single consumer. In each country the representative firm receives an endowment of a single traded good. The only tradable financial assets are the money forward exchange contracts. In addition, there are no contingent claims markets, so all possibilities for reducing risk lie within the forward exchange market, where two maturity contracts are available.

The two consumers own titles to the firms in their respective countries. The timing of the model can be summarized as follows: 1) at the beginning of each period, both firms pay a dividend equal to all incomes gained during the previous period to the respective consumers in their country. The consumer then turns in its dividends for a new money, and the old money becomes worthless. This implies that all the money will be spent; 2) after receiving the money supply, consumers liquidate their forward contracts traded in foreign exchange in the two previous periods, 3) consumers spend their money on the two goods. Domestic goods must be purchased with their own currency. All transactions take place at equilibrium prices. 4) At the end of each period, consumers make forward contracts to deliver currency in the next period.

Let us denote $X_t$ and $X_t^*$ the endowment of goods in the domestic and foreign countries. Endowments of goods are stochastic, and their natural logarithm follows an autoregressive process with a normal innovation.

$$\ln X_t = \mu_X (1 - \rho_X) + \rho_X \ln X_{t-1} + \eta_{X,t}, \quad \eta_{X,t} \sim N\left(0, \sigma_X^2\right) \tag{1}$$

$$\ln X_t^* = \mu_{X^*} (1 - \rho_{X^*}) + \rho_{X^*} \ln X_{t-1}^* + \eta_{X^*,t}, \quad \eta_{X^*,t} \sim N\left(0, \sigma_{X^*}^2\right) \tag{2}$$

We also consider that domestic and foreign good shocks could be correlated. Let $\rho_{XX^*}$ be the correlation coefficient between these shocks.

2.1 The consumer’s problem
The utility function of the home consumer is a CES function:

\[
U_t = \frac{1}{1 - \gamma} \left[ \phi \left( C_{D,t} \right)^\varepsilon + (1 - \phi) \left( C_{F,t} \right)^\varepsilon \right]^{\frac{1 - \gamma}{\varepsilon}}
\]  

where \( C_{D,t} \) and \( C_{F,t} \) are the consumption levels of domestic and foreign goods at time \( t \), \( \gamma \) denotes the relative risk aversion, \( \frac{1}{1 - \gamma} \) is the elasticity of substitution and \( \phi \) is the relative weight of the domestic consumption in the utility function. The more substitutes for any given good, the greater the elasticity will tend to be. The optimization problem for the home consumer is:

\[
\max E_0 \left[ \sum_{t=0}^\infty \beta^t \frac{1}{1 - \gamma} \left[ \phi \left( C_{D,t} \right)^\varepsilon + (1 - \phi) \left( C_{F,t} \right)^\varepsilon \right]^{\frac{1 - \gamma}{\varepsilon}} \right]
\]

s.t.

\[
P_{D,t} C_{D,t} + P_{F,t} S_t C_{F,t} \leq Y_t
\]

\[
Y_t = M_t + T_{t-1} \frac{S_t - F_{t-1}}{F_{t-1}}
\]

where \( P_{D,t} \) and \( P_{F,t} \) are the prices of domestic and foreign goods at time \( t \), \( Y_t \) is the total income in period \( t \), \( S_t \) is the spot exchange rate, \( F_t \) is the futures price and \( T_{t-1} \) is the amount of its currency that the home country sold forward in the previous period. The money supply \( (M_t) \) plus the profit obtained from the futures trading activity equals the total home income. A similar optimization problem can be stated for the foreign consumer:

\[
\max E_0 \left[ \sum_{t=0}^\infty \beta^t \frac{1}{1 - \gamma} \left[ \phi \left( C^*_t \right)^\varepsilon + (1 - \phi) \left( C^*_{F,t} \right)^\varepsilon \right]^{\frac{1 - \gamma}{\varepsilon}} \right]
\]

s.t.

\[
P_{D,t} C^*_t + P_{F,t} S^*_t C^*_{F,t} \leq Y^*_t,
\]

\[
Y_t = M^*_t + T^*_{t-1} \frac{S_t - F_{t-1}}{F_{t-1}}.
\]
2.2 Optimal good choices

In any period $t$ the home consumer chooses levels of $C_{D,t}$ and $C_{F,t}$ that maximize $U_t$ subject to the level of his/her home income. First order conditions for choice of $C_{D,t}$ and $C_{F,t}$ are:

$$\phi \left[ \phi \left( C_{D,t} \right)^\varepsilon + (1-\phi) \left( C_{F,t} \right)^\varepsilon \right]^{-1} \left( C_{D,t} \right)^{\varepsilon - 1} - \lambda_t P_{D,t} = 0, \quad (4)$$

$$\left(1-\phi\right) \phi \left( C_{D,t} \right)^\varepsilon + (1-\phi) \left( C_{F,t} \right)^\varepsilon \right]^{-1} \left( C_{F,t} \right)^{\varepsilon - 1} - \lambda_t S_t P_{F,t} = 0, \quad (5)$$

$$Y_t - P_{D,t} C_{D,t} - P_{F,t} S_t C_{F,t} = 0, \quad (6)$$

where $\lambda_t$ denotes the Lagrange multiplier. From equations (4) and (5), the following relationships are yielded:

$$C_{F,t} = \left[ \frac{\left(1-\phi\right) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t}, \quad (7)$$

where $\sigma = \frac{1}{1-\varepsilon}$ denotes the elasticity of substitution. Using equations (6) and (7), the demand function for the domestic good is as follows:

$$C_{D,t} = \frac{Y_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} \left( 1-\phi \right)^\sigma \left( S_t P_{F,t} \right)^{1-\sigma}}. \quad (8)$$

Substituting equation (8) into equation (7) gives us the demand function for the foreign good:

$$C_{F,t} = \left[ \frac{\left(1-\phi\right) P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma \frac{Y_t P_{D,t}^{-\sigma}}{P_{D,t}^{1-\sigma} \left( 1-\phi \right)^\sigma \left( S_t P_{F,t} \right)^{1-\sigma}}. \quad (9)$$

Similar substitutions lead to the demands for the foreign country, that is:
\[
C_{F,t}^* = \left[ \frac{(1-\phi)P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma C_{D,t}^*
\]
(10)

\[
C_{D,t}^* = \frac{Y^*_t S_t P_{D,t}^{-\sigma}}{P_{D,t}^{-\sigma} + \left( \frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}}.
\]
(11)

Then, substituting equation (11) into equation (10) yields the following expression for the foreign demand in the foreign country:

\[
C_{F,t}^* = \left[ \frac{(1-\phi)P_{D,t}}{\phi P_{F,t} S_t} \right]^\sigma \frac{Y^*_t S_t P_{D,t}^{-\sigma}}{P_{D,t}^{-\sigma} + \left( \frac{1-\phi}{\phi} \right)^\sigma (S_t P_{F,t})^{1-\sigma}}.
\]
(12)

2.3 Forward contracting

As well as the allocation of current resources between the domestic and foreign goods, the home consumers choose the level of the forward position in the forex market. The Euler condition is as follows:

\[
E_t \left[ \lambda_{t+1} \beta^{t+1} \left( \frac{S_{t+1} - F_t}{F_t} \right) \right] = 0
\]
(13)

where \(E_t\) denotes the conditional expectation to the information set available in period \(t\).

From equation (13):

\[
E_t [\lambda_{t+1} S_{t+1}] = E_t [\lambda_{t+1} F_{t+1}],
\]
and taking into account equation (4), the value of the forward exchange rate at time \(t\) consistent with the consumer’s optimal choice of the amount of forward contracting can be expressed as follows:

\[
F_t = \frac{E_t \left[ \frac{\partial U_{t+1}}{\partial C_{F,t+1}} \frac{1}{P_{F,t+1}} \right]}{E_t \left[ \frac{\partial U_{t+1}}{\partial C_{F,t+1}} \frac{1}{P_{F,t+1} S_{t+1}} \right]}.
\]
(14)
A similar expression applies for the foreign country, that is:

\[ F_t = \frac{E_t}{\frac{1}{\partial C_{F,t+1}^* P_{F,t+1}} + \frac{1}{\partial C_{F,t+1}^* P_{F,t+1}^* S_{t+1}}} \]  

(15)

2.4 Equilibrium in the goods market

The world constraints on consumption of the two traded goods in both countries imply that the total endowment of the two goods must be equal to the consumption of each good in the respective countries:

\[ C_{D,t} + C_{D,t}^* = X_{D,t} \]  

(16)

\[ C_{F,t} + C_{F,t}^* = X_{F,t} \]  

(17)

Equilibrium prices of the two goods depend on the home and foreign money supplies as well as their total endowment in each country. Taking into account that a) money is worthless after each period and b) each country’s good can only be purchased with that country’s currency, the following cash-in-advance spending constraints must hold:

\[ P_{D,t} X_{D,t} = M_t \]  

(18)

\[ P_{F,t} X_{F,t} = M_t^* \]  

(19)

In addition, in equilibrium, the following relationship between home and foreign futures market for foreign exchange hold:

\[ T_t = -T_t^*. \]

2.5 The monetary policy

Following Andolfatto et al. (2004), we assume that the natural logarithm of the total supply of the base money at the beginning of period \( t \) comprises two stochastic components: one component which reflects the regime (i.e. the underlying preferences
about the long-run monetary policy), and the second being the short-run error made in 
the control of monetary aggregates by the central bank. For the home country, we have:

\[ \ln M_t = \mu + z_t + u_t , \]

where \( M_t \) denotes the home money supply, \( \mu \) is the average of the natural logarithm of 
the money supply, and \( z_t \) and \( u_t \) denote the regime and transitory component, 
respectively. We also assume that the regime component of the monetary policy tends to 
remain constant for a relatively long time period and a new regime occurs only 
occasionally. Thus, the time evolution of \( z_t \) can be expressed as follows:

\[
\begin{aligned}
z_t &= \begin{cases} 
    z_{t-1}, & \text{with probability } p \\
    g_t, & \text{with probability } (1-p), 
\end{cases} 
\quad g_t \sim N\left(0, \sigma_g^2\right),
\end{aligned}
\]

Parameter \( p \) reflects the expected duration of any given regime, or alternatively, 
the persistence of the regime. Given that \( z_t \) represents the long-run monetary guidelines 
of the central bank governor, it is expected that such persistency would be fairly high. 
Parameter \( \sigma_g^2 \) reflects the size of the regime shift. The transitory component of the 
money growth \( u_t \) is assumed to follow a standard AR(1) specification:

\[ u_t = \delta u_{t-1} + a_t , \]

with \( 0 < \delta < 1 \) and \( a_t \sim N\left(0, \sigma_a^2\right) \). The variable \( u_t \) can be interpreted as the outcome of a 
monetary intervention in financial markets as a reaction to shocks occurring in the world 
economy. In a similar way, the dynamics of the monetary policy of the foreign country is 
described as follows:

\[ \ln M_t^* = \mu^* + z_t^* + u_t^* , \]

\[
\begin{aligned}
z_t^* &= \begin{cases} 
    z_{t-1}^*, & \text{with probability } p^* \\
    g_t^*, & \text{with probability } (1-p^*), 
\end{cases} 
\quad g_t^* \sim N\left(0, \sigma_g^2\right), 
\end{aligned}
\]

\[ u_t^* = \delta u_{t-1}^* + a_t^* , \]
with $0 < \delta^* < 1$ and $a_i^* \sim N\left(0, \sigma_i^2\right)$.

We also consider that domestic and foreign monetary shocks concerning the transitory component ($a_i$ and $a_i^*$) could be correlated. Let $\rho_{aa^*}$ be the correlation coefficient between these shocks.

2.6 Spot exchange rates

Using the budget constraints concerning $Y_i$ and $Y_i^*$, and equations (16) to (19), the analytical expression of the equilibrium spot rate is:

$$S_t = \frac{1 - \phi^c}{\phi} \left( \frac{X_{F,t}}{X_{D,t}} \right)^c M_t \frac{M_i}{M_i^*}.$$

To avoid the implications of Siegel’s Paradox, we use the following definition of the risk premium forward market:

$$rp_{t,t+1} = f_t - E_t(s_{t+1}),$$

where $E_t(\cdot)$ denotes the mathematical expectation conditioned on the set of all relevant information at time $t$, $s_t$ and $f_t$ are the logarithm of the spot and forward exchange rate, respectively.

2.7 Expectations

Let us assume that consumers in the economy know the structural parameters. To generate the expectations of the monetary policy we consider two scenarios: a) complete information and b) incomplete information. In the first case, consumers can distinguish the transitory and persistent components of money supply. For the home country, they forecast the future money supply according to the following expression:

$$E_t \ln M_{t+1} = p_{z_t} + \rho u_t.$$

In the more realistic case of incomplete information, consumers are unable to determine which policy regime applies at any given time, that is, historical realizations
of money supply cannot be broken down by consumers into permanent and transitory drivers.

Given that, unconditionally, the bivariate vectors \((z_t, u_t)\) and \((z_t', u_t')\) are joint normally distributed, Bayes’ rule and the Kalman filter are equivalent updating mechanisms.

For example, in the case of the home country, the observation equation and the state transition equation can be expressed as follows:

\[
y_t = H'\xi_t, \\
\xi_{t+1} = F\xi_t + v_{t+1},
\]

where \(y_t = \ln M_t, \xi_t = \begin{pmatrix} z_t & u_t \end{pmatrix}'\), \(v_t = \begin{pmatrix} N_t & a_t \end{pmatrix}\), \(F = \begin{pmatrix} p & 0 \\ 0 & \delta \end{pmatrix}\) and \(N_t\) is a random variable:

\[
N_{t+1} = \begin{cases} \left(1 - p\right)z_t, & \text{with probability } p \\ g_{t+1} - pz_t, & \text{with probability } (1 - p) \end{cases}
\]

Let us denote the variance-covariance matrix of \(v_t\) as \(Q\), and the one-step-ahead forecasts of the unobserved states, conditional on time \(t\) information set, as \(P_{t+1}\). Conditional on starting values \(\hat{\xi}_{10}\) and \(P_{10}\) the following recursive structure that describes the evolution of \(\hat{\xi}_{t+1}\) and \(P_{t+1}\) emerges (see Hamilton (1994), chapter 13 for a detailed discussion of the Kalman filter):

\[
K_t = FP_{t+1}H\left(H'P_{t+1}H\right)^{-1}, \\
\hat{\xi}_{t+1} = F\hat{\xi}_{t+1} + K_t(y_t - H'\hat{\xi}_{t+1}), \\
P_{t+1} = (F - K_tH')P_{t+1}(F' - HK_t') + Q,
\]

where \(K_t\) is the Kalman gain matrix. Similar expressions apply to the foreign country.
The solution of the model requires the evaluation of highly non-linear expressions, avoiding the possibility of an analytical solution. Appendix 1 provides a detailed explanation of the solution method considered to obtain simulated equilibrium time series of spot and forward exchange rates. Economic agents need to solve a signal-extraction problem of separating two individual components of the money supply. We assume that they face this problem by constructing an optimal forecast based on all the relevant information.

3 THEORETICAL RESULTS

In this section, we present the theoretical results from simulations. Due to the transitory nature of monetary shock $u$, the value of the parameters $\delta$ and $\delta^*$ would be fairly small. In particular, we consider $\delta = \delta^* = 0.1$ in all numerical simulations. Moreover, the discount factor $\beta$ and the relative risk aversion are constant and equal to 0.99 and 1.50, respectively. The weight of each consumer good in the utility function is $\phi = 0.5$ and the correlation between real shocks ($p_{au}$) is equal to 0.90. Finally, the variance of the transitory component of the home money supply $\sigma_g^2$ is equal to $1 \times 10^{-3}$, while the variance of the innovation into the AR(1) process concerning the transitory domestic and foreign transitory shock is $0.8 \times 10^{-3}$.

Since we are aiming to provide new insights into forward forecasting ability, below we only present the most interesting parameterization, namely, that able to reproduce not only the bias for forward markets in foreign exchange but also the anomaly. In terms of the relevant parameter values concerning the monetary policy and real shocks, these scenarios can be summarized in the following table:
<table>
<thead>
<tr>
<th>Parameters</th>
<th>$p$</th>
<th>$p^*$</th>
<th>$\sigma_{\sigma}^2$</th>
<th>$\rho_{\sigma\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation 1</td>
<td>.050</td>
<td>.050</td>
<td>.001</td>
<td>.900</td>
</tr>
<tr>
<td>Simulation 2</td>
<td>.010</td>
<td>.100</td>
<td>.001</td>
<td>.900</td>
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<td>Simulation 3</td>
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<td>.050</td>
<td>.0001</td>
<td>.900</td>
</tr>
<tr>
<td>Simulation 4</td>
<td>.050</td>
<td>.050</td>
<td>.001</td>
<td>-.900</td>
</tr>
<tr>
<td>Simulation 5</td>
<td>.050</td>
<td>.050</td>
<td>.001</td>
<td>.000</td>
</tr>
</tbody>
</table>

Table 1.

To test the unbiasedness of the forward exchange rate to forecast the future spot rate, following Fama (1984), the econometric specification most commonly used in the literature is the following:

$$ s_{t+k} - s_t = \alpha + \beta (f_{t,k} - s_t) + \varepsilon_{t+k}, $$

where $s_{t+k}$ is the log of the nominal exchange rate in time $t+k$, $f_{t,k}$ is the log of the $k$-period forward rate traded in time $t$ and $\varepsilon_{t+k}$ is a random error. The reason we use the above equation essentially lies in the non-stationarity of spot and forward rates. Given that our model generates stationary spot and forward time series, we consider the “level regression”, that is:

$$ s_{t+k} = \alpha + \beta f_{t,k} + \varepsilon_{t+k}. $$

(20)

Figures 1 to 6 depict not only the rolling beta estimates using a moving window of 24 observations under complete and incomplete information, but also the spot and forward time series, and the time evolution of the permanent component of the monetary supply. Table 2 reports the percentage of negative slope estimates under complete information and rational learning².

² The total number of regressions is 176.
Several questions emerge from the above information set. The model proposed can reproduce the bias for the forward exchange rate to predict the future evolution of the spot rate. The transmission of the monetary policy effects between the two countries appears to be a significant factor in explaining departures from the Uncovered Interest Parity. The number of anomalous slope estimates is clearly higher under high positive correlation between the transitory monetary shocks of the home and the foreign country. When transitory monetary shocks become uncorrelated or negatively correlated, the number of anomalies decreases. The first scenario is closer to the reality of the countries analyzed in the literature. Moreover, when monetary policies are “unbalanced” in terms of either the size of the volatility of the transitory shock or the probability of a regime shift, the number of negative slopes tends to be higher. Interestingly enough, when the home and foreign countries apply a similar monetary policy a similar number of anomalies is detected, but also when agents face a signal-extraction problem. The clear difference in the number of anomalies under complete information and rational learning suggests that when investors cannot distinguish which country is leading the monetary policy implementation, the time period required to learn introduces a distorting factor in the UIP condition.
4 CONCLUSIONS

The forward premium puzzle continues to encourage new research to provide a conclusive explanation. The negative correlation between expected exchange rates and interest rate differentials is an anomaly which has no conclusive explanation in the literature. This paper reexamines the analysis of the forward premium bias in forward markets for foreign exchange. The novelty of the paper lies in its reformulation of the Dutton model to allow monetary policy regimes, different weights in the utility function of the consumption goods and the possibility of either home and foreign real shocks or home and foreign transitory monetary shocks to be correlated. The monetary supply is viewed as having two stochastic components: a) a persistent component that reflects the preferences of the central bank regarding the long-run money supply or inflation target, and b) a transitory component that represents the short-lived interventions.

Numerical simulations presented focus on the role of the monetary policy. In particular we consider two scenarios in generating monetary expectations: a) complete information, that is consumers can distinguish the transitory and persistent components of money supply, and b) incomplete information: in this case consumers are unable to determine which policy regime applies at any given time, and they face a signal-extraction problem.

Our numerical simulations suggest that the erroneous forecasting ability of the forward exchange rate is most likely to appear under high positive correlation between the home and the foreign country. Also, the higher number of anomalous slope estimates tends to appear either when monetary policy in the home country is most likely to shift or volatility of transitory monetary shock is relatively higher.

REFERENCES


Appendix 1

This appendix contains the explanation of the solution method used. The optimality conditions give the following analytical expressions:

\[ S_i = \frac{1 - \phi}{\phi} \left( \frac{X_{F,i}}{X_{D,i}} \right) \epsilon M_i \left( \frac{M_i}{M_i^*} \right) \]  \hspace{1cm} (A.1)

\[ P_{D,i} = \frac{M_i}{X_{D,i}} \]  \hspace{1cm} (A.2)

\[ P_{F,i} = \frac{M_i^*}{X_{F,i}} \]  \hspace{1cm} (A.3)

\[ C_{F,i} = \left[ \frac{(1 - \phi) P_{D,i}}{\phi P_{F,i} S_i} \right]^\sigma C_{D,i} \]  \hspace{1cm} (A.4)

\[ C_{F,i}^* = \left[ \frac{(1 - \phi) P_{D,i}}{\phi P_{F,i} S_i} \right]^\sigma C_{D,i}^* \]  \hspace{1cm} (A.5)

\[ C_{F,i} = \left[ \frac{(1 - \phi) P_{D,i}}{\phi P_{F,i} S_i} \right]^\sigma \left[ M_i + T_{i-1} \left( \frac{S_i - F_{i-1}}{F_{i-1}} \right) P_{D,i}^{\sigma} \right] \right] \] \begin{bmatrix} \frac{1 \cdot \phi^\sigma}{\phi^\sigma} \left( S_i P_{F,i} \right)^{1-\sigma} \end{bmatrix} \]  \hspace{1cm} (A.6)

\[ C_{F,i}^* = \left[ \frac{(1 - \phi) P_{D,i}}{\phi P_{F,i} S_i} \right]^\sigma \left[ M_i^* + T_{i-1} \left( \frac{S_i - F_{i-1}}{F_{i-1}} \right) S_i P_{D,i}^{\sigma} \right] \right] \] \begin{bmatrix} \frac{1 \cdot \phi^\sigma}{\phi^\sigma} \left( S_i P_{F,i} \right)^{1-\sigma} \end{bmatrix} \]  \hspace{1cm} (A.7)

\[ T_{i-1}^* = T_{i-1} \]  \hspace{1cm} (A.8)
\[
F_t = \frac{E_t \left[ C_{D,t+1}^{e-1} \left( \phi C_{D,t+1}^{e} + (1 - \phi) C_{F,t+1}^{e} \right)^{1 - \gamma - 1} \frac{1}{P_{D,t+1}} \right]}{E_t \left[ C_{D,t+1}^{e-1} \left( \phi C_{D,t+1}^{e} + (1 - \phi) C_{F,t+1}^{e} \right)^{1 - \gamma - 1} \frac{1}{S_{t+1} P_{D,t+1}} \right]}, \quad (A.9)
\]

\[
F_t = \frac{E_t \left[ \left( C_{F,t+1}^{*} \right)^{e-1} \left( \phi \left( C_{D,t+1}^{*} \right)^{e} + (1 - \phi) \left( C_{F,t+1}^{*} \right)^{e} \right)^{1 - \gamma - 1} \frac{1}{P_{F,t+1}} \right]}{E_t \left[ \left( C_{F,t+1}^{*} \right)^{e-1} \left( \phi \left( C_{D,t+1}^{*} \right)^{e} + (1 - \phi) \left( C_{F,t+1}^{*} \right)^{e} \right)^{1 - \gamma - 1} \frac{1}{S_{t+1} P_{F,t+1}} \right]}, \quad (A.10)
\]

Substituting equations (1) to (8) into equations (9) and (10) leads to the following relationships:

\[
F_t = \frac{E_t \left[ g_1 \left( F_t, T_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1} \right) \right]}{E_t \left[ g_2 \left( F_t, T_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1} \right) \right]}, \quad (A.11)
\]

\[
F_t = \frac{E_t \left[ g_3 \left( F_t, T_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1} \right) \right]}{E_t \left[ g_4 \left( F_t, T_t; M_{t+1}, M_{t+1}^*, X_{D,t+1}, X_{F,t+1} \right) \right]}, \quad (A.12)
\]

where

\[
g_1 = \omega_D^{-\gamma} \left[ \phi + (1 - \phi) \left( \frac{X_{F,t+1}}{X_{D,t+1}} \right)^{1 - \gamma - 1} \left( \frac{X_{D,t+1}}{M_{t+1}} \right), \right]
\]

\[
g_2 = \omega_D^{-\gamma} \left[ \phi + (1 - \phi) \left( \frac{X_{F,t+1}}{X_{D,t+1}} \right)^{1 - \gamma - 1} \left( \frac{X_{D,t+1}}{M_{t+1}} \right) \frac{\phi}{1 - \phi} \left( \frac{X_{D,t+1}}{X_{F,t+1}} \right)^{1 - \gamma - 1} \left( \frac{M_{t+1}^*}{M_{t+1}} \right), \right]
\]

\[
g_3 = \left( \frac{X_{F,t+1}}{X_{D,t+1}} \right)^{1 - \gamma - (\varepsilon - 1)} \omega_D^{-\gamma} \left[ \phi + (1 - \phi) \left( \frac{X_{F,t+1}}{X_{D,t+1}} \right)^{1 - \gamma - 1} \left( \frac{X_{F,t+1}}{M_{t+1}} \right)^{1 - \gamma - 1} \left( \frac{X_{F,t+1}}{M_{t+1}} \right), \right]
\]
Using linear and log-linear approximation of functions \( g_j(\cdot), j = 1, 2, 3, 4 \) and then substituting into equations (A.9) and (A.10), we obtain a system of two equations in two variables \((F_t, T_t)\) as a function of the following expectations:

\[
E_t(\hat{x}_{D,t+1}), E_t(\bar{x}_{F,t+1}), E_t(\hat{m}_{t+1}), E_t(\hat{m}^*_t),
\]

where:

\[
\hat{x}_{D,t+1} \equiv \ln(X_{D,t+1} / X_D), \bar{x}_{D,t+1} \equiv \ln(X_{D,t+1} / X_{D,ss}), \bar{x}_{F,t+1} \equiv \ln(X_{F,t+1} / X_{F,ss}),
\]

\[
\hat{m}_{t+1} \equiv \ln(M_{t+1} / M_{ss}), \bar{m}_{t+1} \equiv \ln(M^*_t / M^*_{ss}),
\]

and \( X_{D,ss}, X_{F,ss}, M_{ss}, M^*_{ss} \) denote the steady-state equilibrium values.

Once the relevant expectations have been computed, the above system can be solved. We want to highlight the fact that agents know the dynamics structure of endowments. However, we consider two scenarios to generate monetary expectations: a)
complete information, that is, agents also know the specification of the dynamics governing the time evolution of the money supply and b) incomplete information, in this case agents face a signal-extraction problem to separate the permanent and transitory components of the money supply. To deal with this issue the Kalman filter is applied.
Appendix 2 Figures

**Figure 1**

- **Spot and forward under incomplete information**
- **Spot and forward under complete information**
- **Forward with rational learning and forward with complete information**
- **Domestic monetary policy shifts**
- **Foreign monetary policy shifts**
- **Domestic and foreign monetary policy shifts**

**ROLLING BETA ESTIMATES**

- Beta estimates with rational learning
- Beta estimates with complete information
Spot and forward under incomplete information

Spot and forward under complete information

Forward with rational learning and forward with complete information

Domestic monetary policy shifts

Foreign monetary policy shifts

Domestic and foreign monetary policy shifts

ROLLING BETA ESTIMATES

Beta estimates with rational learning
Beta estimates with complete information

Figure 2
Figure 3
Figure 4
Figure 5