MINIMIZING POWER CONSUMPTION IN DRYING SYSTEMS PROPELLED BY THERMAL OR SOLAR ENERGY

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Abstract: We consider minimum power associated with consumption of thermal or solar energy which drives various one-stage and multi-stage drying operations. In these evaluations essential role is played by the static optimization and the optimal control theory. We consider both devices of energy generator type (engines) and of heat pump or separator type (energy consumers), each of these devices being driven either by the radiative heat exchange or by the simultaneous transfer of energy and mass. Also, we point out the link between the present irreversible approach and the classical problem of minimum reversible work. Minimization of total power supplied to a multi-stage drying system propelled by the heat pumps is analyzed by the method of dynamic programming.

Keywords: thermal efficiency, solar heat pumps, heat pump dryers, second law.

INTRODUCTION

The evaluation of limiting minimum power supplied to various energy units is important for systems with thermal and solar heat pumps, drying separators and electrolysis units. Yet, we focus here on the power limits arising in various thermal drying systems. Optimization analyses lead to the dryer’s efficiency and limiting power. The developed thermodynamics, implies the limits on the work consumption which are stronger (higher) than those predicted by the classical work of thermodynamics. These limits are used in engineering design. Classical reversible theory is capable of determining energy limits in terms of exergy changes. However, they are too distant from reality (real energy consumption is higher than the lower bound and/or real energy yield is lower than the upper bound). Yet, by introducing rate dependent factors, irreversible thermodynamics offers more realistic limits. In our example here we focus on limits evaluated for the work supplied to a heat pump which heats a drying gas. They lead to estimates of minimum work supplied to a heat pump.

IMPERFECT EFFICIENCIES

By evaluating entropy production in an infinitesimal cycle $\sigma$, (the sum of external and internal parts) as the difference between the outlet and inlet entropy fluxes we find in terms of the first-law efficiency $\eta$

$$dS_e = -\frac{dQ_e}{T_2} - \frac{dQ_1}{T_1} = \frac{dQ_1}{T_2} (1 - \eta - \frac{T_2}{T_1})$$

where $T_1$ and $T_2$ are the temperatures of the two reservoirs, respectively. Equation (1) states that the deviation of engine’s efficiency from the Carnot efficiency is related to the entropy production. This property leads us to an important analytical formula for the real efficiency, suitable in process optimization. To derive the formula we note that the thermal efficiency of any real thermal engine or heat pump can always be written in the form

$$\eta = 1 - \frac{dQ_2}{dQ_1}$$

In terms of the factor of internal irreversibilities $\Phi = 1 + T_1 dQ_{int} / dQ_1$ the entropy balance of working medium takes a form

$$\Phi dQ_1 / T_1 = dQ_2 / T_2$$

The factor $\Phi$ can be found from the internal entropy production within machine. As often $\Phi$ is a complicated function of the operating variables an averaged $\Phi$ over the cycle is used, treated as the process constant. In terms of the unknown temperatures of circulating fluid and the internal irreversibility index $\Phi$ real efficiency $\eta$ follows as

$$\eta = 1 - \frac{dQ_2}{dQ_1} = 1 - \Phi T_2 / T_1$$

This equation simplifies, of course, to the Carnot formula in terms of both primed $T$ when internal entropy source vanishes (the so-called endoreversible operation). Note that no special assumptions were made to derive equation (1) and (4).
Now a quantity called Carnot temperature $T'$ can be introduced that satisfies the thermodynamic relation
\[ T' = T_2 T'_1 / T_2'. \] (5)

The name Carnot temperature is used for the quantity $T'$ simply because the efficiency of an internally reversible engine expressed in terms of $T'$ and $T_2$ satisfies the Carnot formula, see Eq. (6) below. Note that, in agreement with Eq. (5), temperature $T'$ represents the effect of the ratio of temperatures of circulating fluid, $T_1/T_2'$, in a thermal machine. After using Eq. (5) in Eq. (4) the thermal efficiency $\eta$ in terms of temperature $T'$ assumes a simple, pseudo-Carnot form (Sieniutycz 2003)
\[ \eta = 1 - \Phi T'_2 / T' \] (6)
Equation (6) is suitable to evaluate the power production fluxes in steady systems. Yet, to get a heat flux one must apply a model of heat exchange.

**THERMAL AND MECHANICAL POWER**

To enhance the model generality we use here a model with conductances $g_1$ and $g_2$ that are functions of $T_1$ and $T_2$. This allows us to describe nonlinear exchange processes, e.g. those with radiation. Variable heat transfer coefficients are $\alpha_1(T_1)$ and $\alpha_2(T_2)$. The entropy balance for the internal part of the machine
\[ g_2(T_2) (T_2' - T_2) T_2'^{-1} - \Phi g_1(T_1) (T_1 - T_2') T_2'^{-1} = 0 \] (7)
proves that $T'$ is a single unconstrained control $T'$. We use Eq.(5) to insert $T_2' = T_1 T_2 / T'$ into Eq. (7).

We then obtain $T_1'$, in terms of $T'$
\[ T_1' = (\Phi g_1 T_1 + g_2 T') (\Phi g_1 + g_2)^{-1} \] (8)
and the corresponding equation for $T_2' = T_1 T_2 / T'$. The driving energy flux follows as
\[ q_1 = g_1(T_1' - T_1) = g'(T_1 - T') \] (9)
and the flux $q_2 = q_1(1-\eta)$, where $\eta$ is defined by the pseudo-Carnot expression (6). In Eq. (9) an overall conductance, $g'$, appears
\[ g'(\Phi T_1 T_2) = g_1 g_2 (\Phi g_1 + g_2)^{-1} = (g_1^{-1} + \Phi g_2^{-1})^{-1} \] (10)
This is, in fact, a suitably modified overall conductance of an inactive heat transfer in which the use of the operative $(\Phi$ and $T$ dependent) heat conductance, $g'$, is required. From Eq. (6) and (9) the propelling mechanical power $p = \eta q_1$ follows
\[ p = q_1 \eta = g'(\Phi T_1 T_2) (T_1 - T')(1 - \Phi T_2 / T') \] (11)
where $g'$ is defined by Eq. (10). Optimal control $T'$ ensuring the upper bound for the power production is $T'_{\text{opt}} = \Phi T_1 T_2$. $T'_{\text{opt}}$ is also the optimal temperature of a solar collector maximizing its exergy output when $T_1$ and $T_2$ are the collector’s stagnation and ambient temperatures, and $\Phi = 1$. Eq. (11) serves to minimize the work supply in a process with drying agent heated by heat pumps (Berry *et al.*, 2000).

**POWER FOR DRYING WITH HEAT PUMPS**

Let us consider an operation in which a drying agent should be heated before the dryer in order to achieve a sufficiently high temperature. Assume a multistage heating accomplished in the condensers of heat pumps. The multi-stage operation uses data obtained for one stage. The typical state changes of gas in a related multistage system are illustrated on the enthalpy-concentration diagram in Fig. 1.

We assume that a pulverized solid contacts with a gas concurrently, and the drying occurs in the first drying period. The gas leaving the first stage enters the heat pump and dryer of the second stage, similarly the outlet solid from the first stage flows to the dryer of this second stage. The process may be continued. We also assume that the outlet solid and gas are in the equilibrium due to a large specific solid area.

The purpose is to minimize the work consumption in the two-stage operation by a suitable choice of the intermediate moisture content between the first and the second stage. The balances of mass and heat yield
\[ \frac{r G_x}{c G_\alpha} (W^p - W^t) = T'_{\text{opt}} - T', \] (12)
\[ T'_{\text{opt}} - T' = \frac{c}{c} (X'_{\text{opt}} - X_T(T')) , \] (13)
whereas the power consumed at a single stage per unit flow of gas, $e'$, is described by an expression
\[ e' = -\frac{p}{G_\alpha} = c \left( 1 - \frac{T'}{T'_1 + u} \right) q'_v \theta', \] (14)
where $q'_v$ is the energy supply to the drying gas in the condenser of the heat pump, and $u' = - q'_v / \phi > 0$ is a measure of this energy supply in the temperature units. Substituting into the above equation the temperature $T'_1$ following from Eq. (13)
\[ T'_1 = T^d + \frac{c}{c} (X_T(T') - X'_{\text{opt}}) \] (15)
and taking into account that $X'_v = X_{\text{opt}}^p$ [also $X'_v = X_j$ for $n = 2,..N$] we find the mechanical energy consumption at the stage
Fig. 1. Changes of gas states in a multistage work-assisted drying operation. Primed states refer to temperatures of circulating fluids which heat gases supplied to dryers 1, 2...n.

\[ e^1 = c \left( 1 - \frac{T^e}{T^1 + \frac{r}{c} \left( X_s(T^1) - X^0 \right) + u^1} \right) u^1 \beta^1. \]  

(16)

This is transformed further in view of the link between \( u^1 \) and \( \theta^1 \) (consider difference constraint describing \( \Delta T^1 = u^1 \theta^1 \) for \( n=1 \))

\[ e^1 = c \left( 1 - \frac{T^e}{T^1 + c \left( X_s(T^1) - X^0 \right) + u^1} \right) \]

(17)

\( + rc^{-1} \left( X_s(T^1) - X^0 \right) - T^1 \). \)

Analogous function, but with the shifted superscripts, is valid for the second stage

\[ e^2 = c \left( 1 - \frac{T^e}{T^2 + c \left( X_s(T^2) - X^0 \right) + u^2} \right) \]

(19)

\( + rc^{-1} \left( X_s(T^2) - X^0 \right) - T^2 \). \)

The sum of both works yields the total work consumed. This is the thermodynamic cost that should be minimized. For a fixed time \( \tau^1 = \theta^1 + \theta^2 \) there are two free controls: \( \theta^1 \) and \( T^1 \). Thus we can accomplish the power minimization procedure.

The procedure searches for an optimal interstage temperature \( T^I \) and an optimal heat transfer area of the first heat pump \( a^1 \) present in the control variable \( \theta^1 \). The requirement of sufficiently low final moisture content in solid defines the amount of the evaporated moisture per unit time. The optimization can be generalized to the N-stage cascade system.

**CONCLUDING REMARKS**

The example presented shows how to optimize a drying operation with gas heated by a sequence of heat pumps. The optimal transfer areas are close in value, the optimal temperatures constitute an increasing sequence. The optimal work supplied to the two-stage system decreases distinctly with the total transfer area. With some modification, our approach can be extended to more complex configurations.

**NOMENCLATURE**

- \( c \) specific heat of unit volume \([Jm^{-3}K^{-1}]\)
- \( e \) power supply per unit mass flux \([Jkg^{-1}]\)
- \( g, g \) partial and overall conductance \([Js^{-1}K^{-a}]\)
- \( p \) mechanical power output \([Js^{-1}]\)
- \( Q \), heat delivered from the first reservoir \([J]\)
- \( S, S \) total entropy and entropy produced, \([JK^{-1}]\)
- \( T^1, T^2 \) bulk temperatures of reservoirs 1 and 2 \([K]\)
- \( T^1, T^2 \) temperatures of circulating fluid (Fig. 1) \([K]\)
- \( T^C \) Carnot temperature control \([K]\)
- \( \alpha^1 \) overall heat transfer coefficient \([Jm^{-2}s^{-1}K^{-1}]\)
- \( \eta = p/Q_t \) first-law efficiency \([-]\)
- \( \Phi \) factor of internal irreversibility \([-]\)

**REFERENCES**
