Full-Multigrid (FMG): the most efficient multigrid algorithm

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Abstract

In order to accelerate iterative solution procedures, the use of hierarchies of computational grids of various resolutions is a well-known technique. Multigrid algorithms are among this kind of methods, and they have become one of the most efficient numerical techniques for solving the algebraic linear equation systems arising from the discretization of PDEs. By other hand, the obtention of a good initial approximation can benefit the performance of iterative methods. The construction of such an approximation by means of inexpensive computations on coarser grids is known as nested iteration. The combination of nested iteration and multigrid computational techniques yields the so-called full multigrid algorithm. FMG is the most efficient approach to multigrid methods, since it is considered to be asymptotically optimal, that is, the number of arithmetic operations required is proportional to the number of grid points, with only a small constant of proportionality. The goal of the FMG algorithm should be to yield a numerical solution whose error is comparable to the discretization error. Typically, the common lore states that one or two multigrid cycles are sufficient to reach such discretization accuracy. However, the key question then is whether the solution obtained by this algorithm is “sufficiently accurate”, and in practice, it may be quite difficult to assess whether the FMG solution indeed yields discretization-level accuracy. This notion was formalized in [1] by defining a worst-case relative accuracy measure, denoted $E^r_{FMG}$, which compares the total error of the $\ell$-level FMG solution against the inherent discretization error. This measure can be used for tuning algorithmic components so as to obtain discretization-level accuracy. In this work, local Fourier analysis framework for FMG is also developed for estimating $E^r_{FMG}$, resulting in a tool which yields, a-priori, valuable insights into the various components of the FMG algorithm and their effect on the final relative accuracy.

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Bibliography