Asymptotic Analysis of the Eigenvalues of an Elliptic Problem in an Anisotropic Thin Multidomain

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Sesión especial: “Homogeneización y Perturbaciones Espectrales”

This is a joint work with Ali Sili.

For every \( n \in \mathbb{N} \), let \( \Omega_n \subset \mathbb{R}^N \) be a thin multidomain consisting of two vertical cylinders, one placed upon the other: the first one with height 1 and small cross section \( r_n \omega \), the second one with small thickness \( h_n \) and cross section \( \omega \), where \( \omega \) is a bounded open connected regular subset of \( \mathbb{R}^{N-1} \) containing the origin of \( \mathbb{R}^{N-1} \), and \( r_n \) and \( h_n \) are two vanishing positive parameters. Precisely,

\[
\Omega_n = (r_n \omega \times [0,1]) \bigcup (\omega \times ]-h_n,0[).
\]

In \( \Omega_n \) we consider the following eigenvalue problem:

\[
\begin{align*}
-\text{div} (A_n(x) DU_n) &= \lambda U_n \quad \text{in } \Omega_n, \\
U_n &= 0 \quad \text{on } \Gamma_n = (r_n \omega \times \{1\}) \cup (\omega \times ]-h_n,0[), \\
A_n(x) DU_n\nu &= 0 \quad \text{on } \partial\Omega_n \setminus \Gamma_n,
\end{align*}
\]

(0.1)

where \( \nu \) denotes the exterior unit normal to \( \Omega_n \), \( r_n \omega \times \{1\} \) is the top of the upper cylinder, \( \partial \omega \times ]-h_n,0[ \) is the lateral surface of the lower one, and

\[
A_n(x',x_N) = \begin{cases} A \left( \frac{x'}{r_n},x_N \right) & \text{a.e. in } \Omega^a_n = r_n \omega \times ]0,1[, \\
A \left( \frac{x'}{h_n},x_N \right) & \text{a.e. in } \Omega^b_n = \omega \times ]-h_n,0[,
\end{cases}
\]

(0.2)

\( A(x) = (a_{ij}(x))_{i,j=1,...,N} \) being a measurable, bounded, uniformly elliptic and symmetric matrix valued function defined in \( \omega \times ]-1,1[ \). Remark that assumption (0.2) allows to consider different types of materials in \( \Omega_n \). For instance, one can consider a homogeneous isotropic material, or a homogeneous anisotropic material, or a non-homogeneous anisotropic material.

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where the matrix is independent of $x'$ in $\Omega_n^a$ and independent of $x_N$ in $\Omega_n^b$, or a cylinder $\Omega_n^c$ composed of two materials: a cylindrical hearth enveloped by a cylindrical shell made by a different material, etc.

It is well known that, for every $n \in \mathbb{N}$, there exist an increasing diverging sequence of positive numbers $\{\lambda_{n,k}\}_{k \in \mathbb{N}}$ and a $L^2(\Omega_n)$-Hilbert orthonormal basis $\{U_{n,k}\}_{k \in \mathbb{N}}$, such that $\{\lambda_{n,k}\}_{k \in \mathbb{N}}$ forms the set of all the eigenvalues of Problem (0.1) and, for every $k \in \mathbb{N}$, $U_{n,k} \in \mathcal{V}_n = \{V \in H^1(\Omega_n) : V = 0$ on $\Gamma_n\}$ is an eigenvector of (0.1) with eigenvalue $\lambda_{n,k}$. Moreover, $\left\{\frac{\lambda_{n,k}^{-1}}{2} U_{n,k}\right\}_{k \in \mathbb{N}}$ is a $\mathcal{V}_n$-Hilbert orthonormal basis, by equipping $\mathcal{V}_n$ with the inner product:

$$(U, V) \in \mathcal{V}_n \times \mathcal{V}_n \rightarrow \int_{\Omega_n} A_n DUDV dx.$$

The aim of this work is to study the asymptotic behavior, as $n$ diverges, of the sequences $\{(\lambda_{n,k}, U_{n,k})\}_{n \in \mathbb{N}}$, for every $k \in \mathbb{N}$, when $h_n \simeq r_n^{N-1}$, for obtaining a more "handling" eigenvalue problem for the 1D -(N - 1)D limit domains.

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